IIT-JEE 2003 Mains Questions & Solutions - Maths
(The questions are based on memory)

Break-up of questions:

Algebra	Trigonometry	Co-ordinate Geometry	Calculus	Vector/3D
8	1	1	8	2

1. Prove that $\left|\dfrac{1-z_1\bar{z}_2}{z_1-z_2}\right|<1$ if $|z_1| < 1 < |z_2|$

[2]

Sol. T.P.T. $\left|\dfrac{1-z_1\bar{z}_2}{z_1-z_2}\right|<1$

or, T.P.T. $|1-z_1\bar{z}_2| < |z_2 - z_1|$

or, T.P.T. $(1-z_1\bar{z}_2)(1-\bar{z}_1 z_2) < (z_2 - z_1)(\bar{z}_2 - \bar{z}_1)$

or, T.P.T. $1+|z_1|^2|z_2|^2 - |z_1|^2 - |z_2|^2 < 0$

or, T.P.T. $(1-|z_1|^2)+(|z_1|^2-1)|z_2|^2<0$

or, T.P.T. $(1-|z_1|^2)(1-|z_2|^2)<0$

Which is true because of $|z_1| < 1 < |z_2|$

2. P(x) is a polynomial function such that P(1) = 0, P′(x) > P(x), ∀ x > 1. Prove that P(x) > 0, ∀ x > 1

[2]

Sol. $P'(x) - P(x) > 0,\ \forall x > 1$

⇒ $e^{-x}.P'(x) - e^{-x}P(x) > 0$, ∀ x > 1 (multiplying by e^{-x} which is +ve)

⇒ $\dfrac{d}{dx}\left(e^{-x}.p(x)\right) > 0,\ \forall x > 1$

⇒ $e^{-x}.P(x)$ is an increasing function of x, ∀ x ∈ [1, ∞) (as P(x) being polynomial function is a continuous function).

Thus for x > 1

⇒ $e^{-x}.P(x) > e^{-1}.P(1)$

⇒ $e^{-x}.P(x) > 0$, as P(1) = 0

⇒ P(x) > 0. (as e^{-x} +ve)

3. If $A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, abc =1, $A^T A = I$, then find the value of $a^3 + b^3 + c^3$.

[2]

Sol. $A^T A = I$

$\Rightarrow$ (det A) (det A^T) = 1 $\Rightarrow$ (det A)^2 = 1 $\Rightarrow$ det A = ± 1.

Now det A = –($a^3 + b^3 + c^3$ – 3abc) = –($a^3 + b^3 + c^3$) + 3

Thus $-(a^3+b^3+c^3)+3=\pm 1 = k(say)$ (say)

$\Rightarrow$ $a^3 + b^3 + c^3$ = 3 – k = 2 or 4.

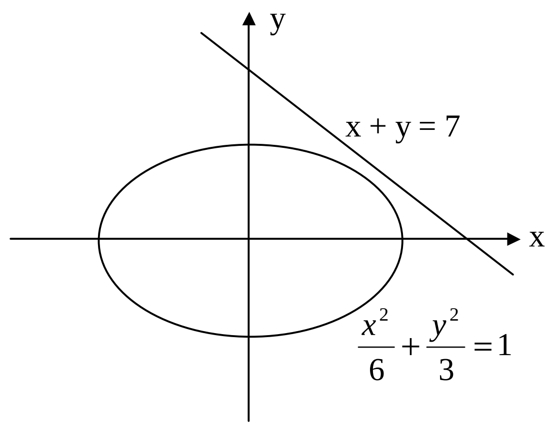
4. Find the point on $x^2 + 2y^2 = 6$, which is nearest to the line x + y = 7.

[2]

Sol. Let P ≡ ($\sqrt{6}$ cosθ, $\sqrt{3}$ sinθ) be the required point

Tangent at P should be of slope = –1 (slope of the line x + y = 7)

Now $x^2 + 2y^2 = 6$



$\Rightarrow$ $x + 2y\frac{dy}{dx} = 0$

$\Rightarrow$ $\frac{dy}{dx} = -\frac{x}{2y}$

$\Rightarrow$ $\left[\frac{dy}{dx}\right]_P = -\frac{\sqrt{6}\cos\theta}{2\sqrt{3}\sin\theta} = -\frac{1}{\sqrt{2}}\cot\theta = -1$

$\Rightarrow$ cotθ = $\sqrt{2}$

$\Rightarrow$ sinθ = $\frac{1}{\sqrt{3}}$, cosθ = $\sqrt{\frac{2}{3}}$ (as P lies in the I'st quadrant)

Then $P \equiv \left(\sqrt{6}.\frac{\sqrt{2}}{\sqrt{3}}, \sqrt{3}.\frac{1}{\sqrt{3}}\right) \equiv (2, 1)$

5. Prove that $2^k\binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{1}\binom{n-1}{k-1}.......(-1)^k\binom{n}{k}\binom{n-k}{0} = \binom{n}{k}$ where $\binom{n}{m} = {}^nC_m$.

[2]

Sol. $2^k\binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{1}\binom{n-1}{k-1}...(-1)^k\binom{n}{k}\binom{n-k}{0}$

= coefficient of x^k in $\left[{}^nC_0(1+2x)^n - {}^nC_1 x(1+2x)^{n-1} + ...\right]$

= coefficient of x^k in $[(1+2x)-(x)]^n$
= coefficient of x^k in $(1 + x)^n$
= ${}^nC_k$.

6. If $|a_i| < 2$, $I \in \{1, 2, 3\ldots\ldots, n\}$. Prove that for no z, $|z| < \frac{1}{3}$ and $\sum_{i=1}^{n} a_i z^i = 1$ can occur simultaneously.

[2]

Sol. $1 = \left|\sum a_i Z^i\right| \le \sum |a_i Z^i|$

$\Rightarrow \quad 1 \le |a_1 Z| + |a_2 Z^2| + |a_3 Z^3| + \ldots\ldots + |a_n Z^n|$

$< 2\left(|Z| + |Z|^2 + |Z|^3 + \ldots\ldots + |Z|^n\right)$

$\Rightarrow \quad 1 + |Z| + |Z|^2 + |Z|^3 + \ldots.. + |Z|^n > 3/2$

Case I

$|Z| < 1$

$\Rightarrow \quad 1 + |Z| + |Z|^2 + \ldots.. \infty > 3/2$

$\Rightarrow \quad \frac{1}{1-|Z|} > \frac{3}{2}$

$\Rightarrow \quad 2 > 3 - 3|Z|$

$\Rightarrow \quad |Z| > 1/3$

Case II

$|Z| \ge 1$, then
obviously, $|Z| < 1/3$ is not possible

Hence $|Z| < 1/3$ and $\sum_{i=1}^{n} a_i z^i = 1$ can not occur simultaneously for any a_i, $|a_i| < 2$.

7. If $f : [-2a, 2a] \to R$ be an odd function such that left hand derivative at x = a is zero and f(x) = f(2a – x), $x \in (a, 2a)$, then find left hand derivative of f at x = –a.

[2]

Sol. $f'(a-) = \underset{h\to 0^-}{Lt} \frac{f(a+h) - f(a)}{h} = 0$, $x \in (0, 2a)$

Now $f'(-a-) = \underset{h\to 0^-}{Lt} \frac{f(-a+h) - f(-a)}{h}$

$= \underset{h\to 0^-}{Lt} \frac{-f(a-h) + f(a)}{h}$, f is an odd function.

$$= \lim_{h\to 0^-} \frac{-f(a+h)+f(a)}{h}, \; f(x) = f(2a - x),\; x \in (a, 2a)$$

$$= -\lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h} = 0$$

8. If f(x) is an even function, then prove that

$$\int_0^{\pi/2} f(\cos 2x)\cos x dx = \sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x dx$$

[2]

Sol.

$$I = \int_0^{\pi/2} f(\cos 2x)\cos x\; dx$$

$$I = \int_0^{\pi/2} f(\cos 2(\pi/2 - x))\cos(\pi/2 - x)dx$$

$$I = \int_0^{\pi/2} f(\cos 2x)\sin x dx$$

, as f is even

$$2I = \int_0^{\pi/2} f(\cos 2x)(\cos x + \sin x)dx$$

$$= \sqrt{2}\int_0^{\pi/2} f(\cos 2x)\sin(x + \pi/4)dx$$

$$= \sqrt{2}\int_{-\pi/4}^{\pi/4} f(\cos(\pi/2 + 2t))\cos(t)dt, \quad x + \frac{\pi}{4} = \frac{\pi}{2} + t$$

$$= \sqrt{2}\int_{-\pi 4}^{\pi/4} f(\sin 2t)\cos t dt$$

$$I = \sqrt{2}\int_0^{\pi/4} f(\sin 2t)\cos t dt$$

$$I = \sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x dx$$

.

9. A person has to go through three successive tests. Probability of his passing first exam is P. Probability of passing successive tests is P or P/2 according as he passed the last test or not. He is selected if he passes at least two tests. Find the probability of his selection.

[2]

Sol. Person is selected if either he passes all the tests or exactly two of the tests.
P (passing all the tests) = P.P.P = P^3
Probability of passing two tests
= P(first two tests) + P(first and third tests) + P(second and third tests)

$$= P.P.(1-P) + P.(1-P).\frac{P}{2} + (1-P)\frac{P}{2}P$$

$$= P^2(1-P) + \frac{1}{2}P^2(1-P) + \frac{1}{2}P^2(1-P)$$

$$= 2P^2(1-P)$$

Thus required probability = $P^3 + 2P^2(1-P) = 2P^2 - P^3$.

10. In a combat between *A*, *B* and *C*, *A* tries to hit *B* and *C*, and *B* and *C* try to hit *A*. Probability of A, B and C hitting the targets are 2/3, 1/2, and 1/3 respectively. If A is hit, find the probability that *B* hits *A* and *C* does not.

[2]

Sol. The required probability is given by

$$P(BC' \mid A) = \frac{P(A \mid BC').P(BC')}{P(A \mid BC')P(BC') + P(A \mid B'C).P(B'C) + P(A \mid BC).P(BC) + P(A \mid B'C').P(B'C')}$$

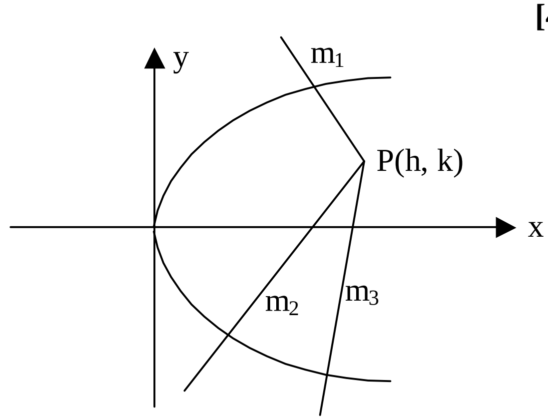
$$= \frac{1.\frac{1}{2}\times\frac{2}{3}}{1.\frac{1}{2}\times\frac{2}{3} + 1.\frac{1}{2}\times\frac{1}{3} + 1.\frac{1}{2}\times\frac{1}{3} + 0\times\frac{1}{2}\times\frac{2}{3}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}+\frac{1}{6}} \qquad = \frac{1}{2}$$

11. Three normals with slopes m_1, m_2 and m_3 are drawn from a point P not on the axis of the parabola $y^2 = 4x$. If $m_1m_2 = \alpha$, results in the locus of P being a part of the parabola, find the value of α.

[4]

Sol. Any normal of slope m to the parabola
$y^2 = 4x$ is
$y = mx - 2m - m^3$ (1)
If it passes through (h, k), then
$k = mh - 2m - m^3$
$\Rightarrow \quad m^3 + (2-h)m + k = 0$ (2)
Thus $m_1m_2m_3 = -k$.



Now $m_1m_2 = \alpha \Rightarrow m_3 = \frac{-k}{\alpha}$
Now m_3 satisfies (2), so

$$-\frac{k^3}{\alpha^3} - (2-h)\frac{k}{\alpha} + k = 0$$

$\Rightarrow \quad k^3 + (2-h)k\alpha^2 - k\alpha^3 = 0$

Thus locus of P is

$$y^3 + (2 - x)y\alpha^2 - y\alpha^3 = 0$$

$\Rightarrow$ $y^2 + (2 - x)\alpha^2 - \alpha^3 = 0$, as $y \neq 0$ (P does not lie on the axis of the parabola)

$\Rightarrow$ $y^2 = \alpha^2 x - 2\alpha^2 + \alpha^3$

If it is a part of the parabola $y^2 = 4x$, then $\alpha^2 = 4$ and $-2\alpha^2 + \alpha^3 = 0$

$\Rightarrow$ $\alpha = 2$

12. Let $f : [0, 4] \rightarrow R$ be a differentiable function

(i) For some a, b $\in$ (0, 4), show that $f^2(4) - f^2(0) = 8f(a).f'(b)$

(ii) Show that $\int_0^4 f(t)dt = 2\left[\alpha f(\alpha^2) + \beta f(\beta^2)\right]$, for some $0 < \alpha; \beta < 2$.

[4]

Sol. (i) Using mean value theorem, there exists b $\in$ (0, 4) such that

$$f'(b) = \frac{f(4) - f(0)}{4} \qquad (1)$$

Now $\left(f(4)\right)^2 - \left(f(0)\right)^2 = \frac{\left(f(4) - f(0)\right)}{4}\left(f(4) + f(0)\right) \times 4$

From (1)

$$\left(f(4)\right)^2 - \left(f(0)\right)^2 = f'(b)\left(f(4) + f(0)\right) \times 4$$

Hence it is sufficient to prove that

$$\frac{f(0) + f(4)}{2} = f(a)$$

Range of function f must contain the interval $[f(0), f(4)]$ or $[f(4), f(0)]$ according as $f(0) \leq f(4)$ or $f(0) \geq f(4)$

Now $\frac{f(0) + f(4)}{2}$ is the mean value of $f(0)$ and $f(4)$

$\Rightarrow$ $\left(\frac{f(0) + f(4)}{2}\right) \in$ range of the function

$\Rightarrow$ a $\in$ [0, 4] for which $f(a) = \frac{f(0) + f(4)}{2}$. Hence proved.

(ii) Let $\sqrt{t} = x \Rightarrow t = x^2 \Rightarrow dt = 2x\ dx$.

Thus $\int_0^4 f(t)dt = 2\int_0^2 xf(x^2)dx = 2(2 - 0)f(\varepsilon)$ for some $\varepsilon \in (0, 2)$, (using mean value theorem for definite integral of a differentiable function).

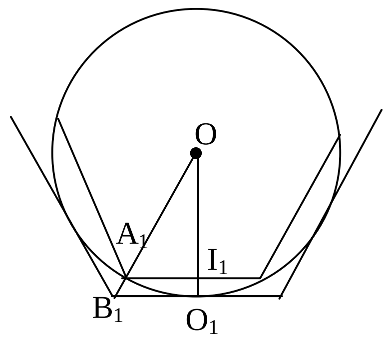
Thus $\int_0^4 f(t)dt = 2\left(f(\varepsilon) + f(\varepsilon)\right)$

$= 2\left(\alpha f(\alpha^2) + \beta f(\beta^2)\right)$, where $\alpha = \beta = \varepsilon$.

13. If I_n represents area of n–sided regular polygon inscribed in a unit circle and O_n the area of the n–sided regular polygon circumscribing it, prove that $I_n = \frac{O_n}{2}\left[1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right]$

[4]

Sol. $I_n = 2n \times$ area of ΔOA_1I_1



$\angle A_1OI_1 = \pi/n$

$= 2n \times \frac{1}{2} \times A_1I_1 \times OI_1$

$= n \times \sin\frac{\pi}{n} \times \cos\frac{\pi}{n}$

$= \frac{n}{2}\sin\frac{2\pi}{n}$.

$O_n = 2n \times$ area of $\Delta\, OB_1O_1$

$= 2n \times \frac{1}{2} \times B_1O_1 \times O_1O$

$= n \times \tan\frac{\pi}{n} \times 1 \qquad = n\tan\frac{\pi}{n}$

Now R.H.S. = $\frac{O_n}{2}\left[1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right]$

$= \frac{O_n}{2}\left[1+\sqrt{1-\sin^2\frac{2\pi}{n}}\right] \qquad = \frac{O_n}{2}\left[1+\cos\frac{2\pi}{n}\right]$

$= \frac{O_n}{2} \times 2\cos^2\frac{\pi}{n} \qquad = O_n.\cos^2\frac{\pi}{n}$

$= n\tan\frac{\pi}{n}.\cos^2\frac{\pi}{n} \qquad = \frac{n}{2}\sin\frac{2\pi}{n} \qquad = I_n$.

Hence proved

14. Find the equation of the plane passing through (2, 1, 0); (4, 1, 1); (5, 0, 1). Find the point Q such that its distance from the plane is equal to the distance of point P(2, 1, 6) from the plane and the line joining P and Q is perpendicular to the plane.

[4]

Sol. Let equation of the plane be

$ax + by + cz + d = 0 \qquad (1)$

(1) passes through the points (2, 1, 0); (4, 1, 1); (5, 0, 1)

$a = -d/3$; $b = -d/3$; $c = \frac{2}{3}d$

$x + y - 2z - 3 = 0$ (2)

which is the required equation of the plane

obviously Q is the image of P in the plane. It is easy to see that Q ≡ (6, 5, –2)

15. If $\hat{u}, \hat{v}, \hat{w}$ be three non–coplanar unit vectors with angles between $\hat{u}$ and $\hat{v}$ is α, between $\hat{v}$ and $\hat{w}$ is β and between $\hat{w}$ and $\hat{u}$ is γ. If $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors along angle bisectors of α, β, γ respectively, then prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right] = \frac{1}{16}\left[\hat{u}\ \hat{v}\ \hat{w}\right]^2 \sec^2\left(\frac{\alpha}{2}\right)\sec^2\left(\frac{\beta}{2}\right)\sec^2\left(\frac{\gamma}{2}\right)$.

[4]

Sol. $\vec{a} = \frac{(\hat{u}+\hat{v})}{2|\cos\alpha/2|}$

$\vec{b} = \frac{(\hat{v}+\hat{w})}{2|\cos\beta/2|}$

$\vec{c} = \frac{(\hat{w}+\hat{u})}{2|\cos\gamma/2|}$

∴ $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right] = \left[\vec{a},\ \vec{b},\ \vec{c}\right]^2 = \left[\frac{((\hat{u}+\hat{v})\times(\hat{v}+\hat{w})).(\hat{w}+\hat{u})}{8|\cos\alpha/2.\cos\beta/2.\cos\gamma/2|}\right]^2$

$= \frac{[\hat{u}\ \hat{v}\ \hat{w}]^2}{16}\sec^2(\alpha/2).\sec^2(\beta/2).\sec^2(\gamma/2)$

16. If a, b and c are in arithmetic progression and a^2, b^2 and c^2 are in Harmonic progression, then prove that either a = b = c or a, b and –c/2 are in Geometric Progression.

[4]

Sol. Given that $2b = a + c$ (1)

a^2, b^2, c^2 are in H.P.

and $b^2 = \frac{2a^2c^2}{a^2+c^2}$ (2)

From (2) $b^2 = \frac{2a^2c^2}{4b^2-2ac}$, using (1)

⇒ $(ac - b^2)(ac + 2b^2) = 0$

⇒ $b^2 = ac$ or $2b^2 = -ac$.

Case I: $b^2 = ac$

⇒ $\left(\frac{a+c}{2}\right)^2 = ac$, using (1)

⇒ $a = c$

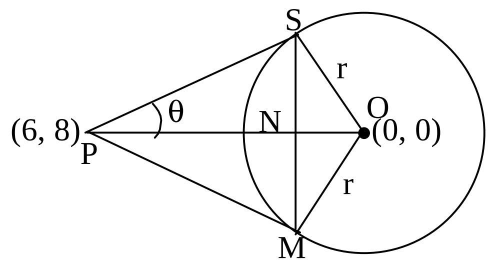
⇒ $a = b = c$, as a, b, c are in A.P.

Case II: $2b^2 = -ac$

⇒ a, b, –c/2 are in G.P. (one of the possibilities)

17. Tangents are drawn from P(6, 8) to the circle $x^2 + y^2 = r^2$. Find the radius of the circle such that the area of the Δ formed by tangents and chord of contact is maximum.

[4]



Sol. $\tan\theta = \frac{r}{PS}$

$= \frac{r}{\sqrt{100 - r^2}}$

$\sin\theta = \frac{r}{10}$

$\cos\theta = \frac{\sqrt{100 - r^2}}{10}$

Area of ΔPSM = $\frac{1}{2}$ SM × PN

$= \frac{1}{2}.2SN \times PN = SN \times PN$

$= SP\sin\theta \times SP\cos\theta$

$= \left(\sqrt{100 - r^2}\right)^2 \times \sin\theta\cos\theta$

$= \left(100 - r^2\right)\frac{r}{10}\frac{\sqrt{100 - r^2}}{10}$

$= \frac{r\left(100 - r^2\right)^{3/2}}{100}$

$\frac{d\Delta}{dr} = \frac{1}{100} \cdot \frac{3}{2}\left(100 - r^2\right)^{1/2}(-2r)r + \frac{\left(100 - r^2\right)^{3/2}}{100} = 0$

$\left(100 - r^2\right)^{1/2}\left(-3r^2 + 100 - r^2\right) = 0$, $r \neq 10$ as P is outside the circle.

$4r^2 = 100 \Rightarrow r^2 = 25 \Rightarrow r = 5$

Thus for r = 5, Δ would be maximum.

18. $x^2 + (a - b)x + (1 - a - b) = 0$, $a, b \in R$. Find the condition on *a*, for which both roots of the equation are real and unequal.

[4]

Sol. For real and unequal roots, $D > 0$

$(a - b)^2 - 4(1 - a - b) > 0, \forall b \in R$

⇒ $b^2 - 2ab + 4b + a^2 + 4a - 4 > 0, \forall b \in R$

$\Rightarrow \quad b^2 - 2(a-2)b + a^2 + 4a - 4 > 0, \forall\, b \in R$

$\Rightarrow \quad 4(a-2)^2 - 4(a^2 + 4a - 4) < 0$

$\Rightarrow \quad a^2 - 4a + 4 - a^2 - 4a + 4 < 0$

$\Rightarrow \quad 8a > 8 \Rightarrow a > 1.$

19. Using $2(1 - \cos x) \le x$, $\forall x \in [0, \pi/4]$ or otherwise prove that $\sin(\tan x) \ge x$, $\forall x \in [0, \pi/4]$

[4]

Sol. Let $f(x) = \sin x - \tan^{-1}x$

$\Rightarrow \quad f'(x) = \cos x - \dfrac{1}{1+x^2}$

Now in the first quadrant cosx is concave down and $\dfrac{1}{1+x^2}$ is concave up,

hence $f'(x) \ge 0$.

Thus f is an increasing function.

Hence $f(x) \ge f(0)$, $\forall\ x \ge 0, x \le 1$

$\Rightarrow \quad \sin x \ge \tan^{-1}x.$

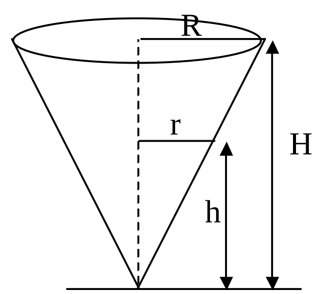
on replacing x by tanx, we get

$\sin(\tan x) \ge x$.

Hence proved.

20. An inverted cone of height H, and radius R is pointed at bottom. It is filled with a volatile liquid completely. If the rate of evaporation is directly proportional to the surface area of the liquid in contact with air (constant of proportionality k > 0). Find the time in which whole liquid evaporates.

[4]



Sol. $\dfrac{R}{H} = \dfrac{r}{h}$

$h = \dfrac{Hr}{R}$

$\dfrac{dv}{dt} = -k\pi r^2$

$\Rightarrow \quad \dfrac{d}{dt}\left(\dfrac{1}{3}\pi r^2 h\right) = -k\pi r^2$

$\Rightarrow \quad \dfrac{d}{dt}\left(\dfrac{r^3 H}{R}\right) = -3Kr^2$

$\Rightarrow \quad \dfrac{dr}{dt} = -k\dfrac{R}{H}$

$\int_R^0 dr = -\dfrac{kR}{H}\int_0^t dt$

$$\Rightarrow \quad -R = \frac{-KR}{H}t \qquad \Rightarrow \quad t = \frac{H}{k}$$

IIT-JEE 2003 Mains Questions & Solutions - Physics
(The questions are based on memory)

Break-up of questions:

Mechanics	Sound	Heat	Electromagnetism	Optics	Modern Physics
6	**2**	**2**	**5**	**2**	**3**

1. *N* divisions on the main scale of a vernier callipers coincide with *N* + 1 divisions on the vernier scale. If each division on the main scale is of *a* units, determine the least count of the instrument.

[2]

Sol. Least count of vernier callipers = value of one division of main scale - value of one division of vernier scale

Now $N \times a = (N+1)a'$ {a' = value of one division of vernier scale)

$$a' = \frac{N}{N+1}a$$

$\therefore$ Least count = a - a' = $\frac{a}{N+1}$

2. Characteristic X-rays of frequency 4.2×10^{18} Hz are produced when transitions from *L* shell to *K* shell take place in a certain target material. Use Mosley's law to determine the atomic number of the target material. Given Rydberg constant $R = 1.1 \times 10^7\ m^{-1}$.

[2]

Sol. According to Bohr's model

$$\Delta E = \lambda v = Rhc(z-b)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \quad \text{for } L \to K$$

$$\sqrt{v} = \sqrt{\frac{3Rc}{4}}(z-1) \qquad (b = 1)$$

$$\sqrt{4.2\times10^{18}} = \sqrt{\frac{3\times1.1\times10^7\times3\times10^8}{4}}(z-1)$$

Solving $(z-1) = 41.194$

$z \approx 42$

3. In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound in air at room temperature.

[2]

Sol. $$\frac{V}{4[L+0.6r]} = 480$$

$\therefore$ $V = 480 \times 4 \times [0.16 + 0.6 \times 0.025] = 336$ m/s

4. An insulated box containing a monatomic gas of molar mass M moving with a speed v_o is suddenly stopped. Find the increment in gas temperature as a result of stopping the box.

[2]

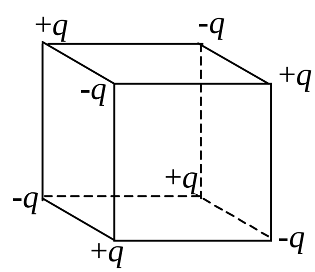
Sol. Decrease in kinetic energy = Increase in internal energy.

$$\frac{1}{2}mv^2 = \frac{m}{M}\frac{R}{\gamma - 1}\Delta T$$

$$\frac{1}{2}v^2 = \frac{1}{M}\frac{3}{2}R\Delta T$$

$$\Delta T = \frac{Mv^2}{3R}$$

5. Eight point charges are placed at the corners of a cube of edge a as shown in the figure. Find the work done in disassembling this system of charges.



[2]

Sol. For potential energy total number of charge pairs = 28.

Let U_1 = Potential energy of charge pairs with separation a (12 pairs)

U_2 = Potential energy of charge pairs with separation $\sqrt{2}a$ (12 pairs)

U_3 = Potential energy of charge pairs with separation $\sqrt{3}a$ (4 pairs)

$$U_1 = \frac{-12Kq^2}{a} \qquad \left[K = \frac{1}{4\pi\varepsilon_o}\right]$$

$$U_2 = \frac{12Kq^2}{\sqrt{2}a}$$

$$U_3 = \frac{-4Kq^2}{\sqrt{3}a}$$

U = total potential energy of system = $U_1 + U_2 + U_3$

$$= \frac{Kq^2}{a}\left[-12 + \frac{12}{\sqrt{2}} - \frac{4}{\sqrt{3}}\right] = -5.824\frac{Kq^2}{a}$$

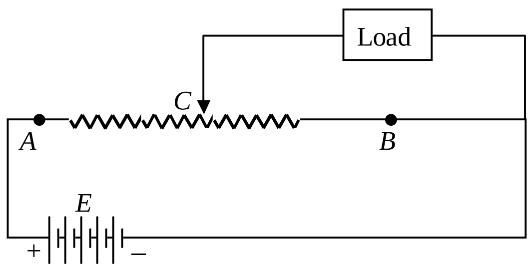
When the charges are separated to infinity, potential energy $U_{\infty} = 0$

Change in energy = 0 - (-5.824 $\frac{Kq^2}{a}$) = 5.824 $\frac{Kq^2}{a}$

So work done by external force = change in potential energy = 5.824 $\frac{Kq^2}{a}$

6. Show by diagram, how can we use a rheostat as the potential divider. **[2]**

Sol.



7. A radioactive element decays by β emission. A detector records n beta particles in 2 seconds and in next 2 seconds it records 0.75n beta particles. Find mean life correct to nearest whole number. Given ln |2| = 0.6931, ln |3| = 1.0986. **[2]**

Sol. $N = N_o e^{-\lambda t}$

$N_2 = N_o e^{-2\lambda}$

and $N_4 = N_o e^{-4\lambda}$

∴ $n = N_o - N_2 = N_o\left(1 - e^{-2\lambda}\right)$

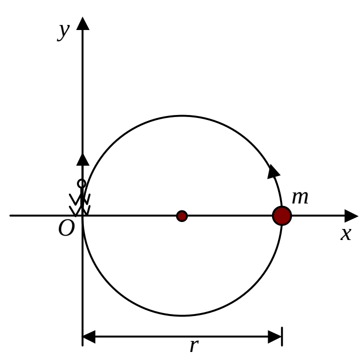
and $0.75n = N_o\left(e^{-2\lambda} - e^{-4\lambda}\right)$

Solving we get,

$\lambda = 0.145\, s^{-1}$

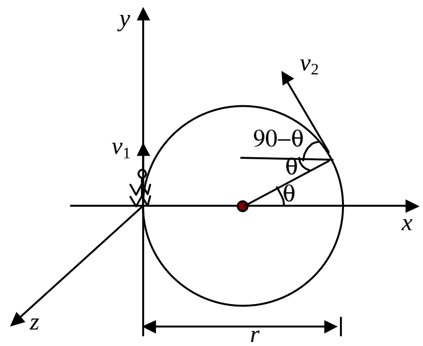
Average life = $\frac{1}{\lambda}$ = 6.896 second.

8. A man and a mass m are initially situated on the diametrically opposite ends as shown in the figure. At some instant they start moving with constant speeds v_1 and v_2. If the man moves in $\hat{j}$ direction and mass moves in a circle of radius r as shown in the figure.



Find the linear momentum of mass with respect to man as a function of time. **[2]**

Sol. Let at any time t mass is situated as shown.



$$\vec{v}_2 = -v_2 \sin\theta \hat{i} + v_2 \cos\theta \hat{j}$$

$$= -v_2 \sin\left(\frac{2v_2}{r} t\right)\hat{i} + v_2 \cos\left(\frac{2v_2}{r} t\right)\hat{j}$$

$$\because \quad v_2 = \frac{r}{2}\omega$$

Relative velocity of the mass with respect to the person is

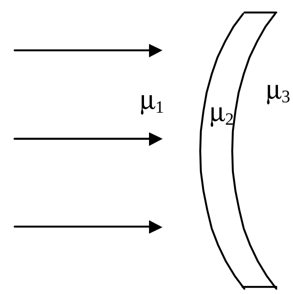
$$= \vec{v}_2 - \vec{v}_1$$

$$= -v_2 \sin\left(\frac{2v_2}{r}.t\right)\hat{i} + \left(v_2 \cos\left(\frac{2v_2}{r}.t\right) - v_1\right)\hat{j}$$

Relative momentum of mass with respect to the person will be

$$m_2\left(-v_2 \sin\left(\frac{2v_2}{r}.t\right)\hat{i} + \left(v_2 \cos\left(\frac{2v_2}{r}.t\right) - v_1\right)\hat{j}\right)$$

9. In the figure, light is incident on the thin lens as shown. The radius of curvature for both the surfaces is R. Determine the focal length of this system.



[2]

Sol. For refraction at 1'st surface,

$$\frac{\mu_2}{\mu_1 v_1} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\frac{1}{R}$$

$$u = \infty$$

$$\therefore \quad \frac{1}{v_1} = \frac{(\mu_2 - \mu_1)}{\mu_2.R} \qquad (1)$$

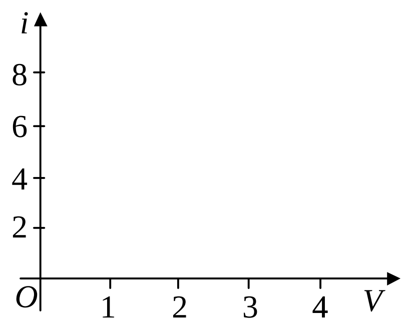
For refraction at 2'nd surface,

$$\frac{\mu_3}{\mu_2.v} - \frac{1}{v_1} = \left(\frac{\mu_3 - \mu_2}{\mu_2.R}\right)$$

Now $v = f$, by putting the value of v_1 from (1)

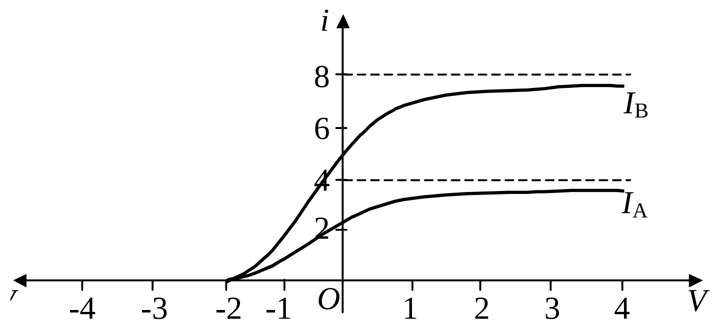
$$\therefore \quad f = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

10. In a photoelectric effect experiment, photons with kinetic energy = 5 eV are incident on a metal surface having work function 3 eV. For intensity of incident photons $I_A = 10^5$ W/m^2 saturation current of 4 μA is obtained. Sketch the graph between i and anode voltage for I_A and $I_B = 2I_A$.

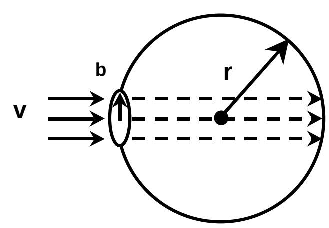


[2]

Sol. Since doubling the intensity doubles the number of photoelectrons, therefore the saturation current will be doubled in the later case.



11. A soap bubble is being blown at the end of a very narrow tube of radius b. Air (density ρ) moves with a velocity v inside the tube and comes to rest inside the bubble. The surface tension of the soap solution is T. After some time the bubble, having grown to a radius r, separates from the tube. Find the value of r. Assume that $r >> b$ so that your can consider the air to be falling normally on the bubble's surface.

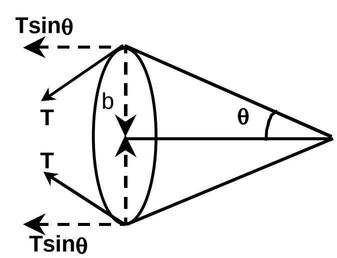


[4]

Sol. Surface Tension force = $2\pi b \times 2T \sin\theta$

Mass of the air per second entering the bubble = $\rho A v$

Momentum of air per second = Force due to air = $\rho A v^2$



The bubble will separate from the tube when force due to moving air becomes equal to the surface tension force inside the bubble.

$$2\pi b \times 2T \sin\theta = \rho A v^2$$

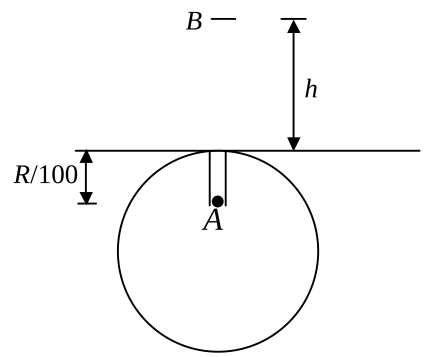
putting $\sin\theta = \frac{b}{r}, A = \pi b^2$ and solving we get

$$r = \frac{4T}{\rho v^2}$$

12. There is a crater of depth $\frac{R}{100}$ on the surface of the moon (radius R). A projectile is fired vertically upward from the crater with a velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile.

[4]

Sol. Let particle be projected from A and it reaches to point B at a height h from surface.



It is projected with escape velocity

$$= \sqrt{\frac{2GM}{R}}$$

Energy at A = energy at B.

K. E. + P. E. (at A) = P. E. at B

$$\frac{1}{2}m\frac{2GM}{R} - \frac{GmM}{2R^3}\left[3R^2 - \left(\frac{99}{100}\right)^2 R^2\right] = -\frac{GMm}{(R+h)}$$

$$\frac{1}{2}m\frac{2GM}{R} - \frac{2.0199GmM}{2R} = -\frac{GMm}{(R+h)}$$

$$-\frac{0.0199GMm}{2R} = -\frac{GMm}{(R+h)}$$

$$R + h = \frac{2R}{0.0199} = 100.5R$$

$$h = 99.5R$$

13. A positive point charge q is fixed at origin. A dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along positive x-axis. Find

(a) the kinetic energy of the dipole when it reaches a distance d from the origin, and

(b) the force experienced by the charge q at this moment.

[4]

Sol.(a) $U = \frac{-1}{4\pi\varepsilon_o}\frac{q}{x^2}p$

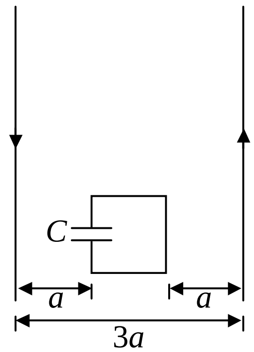
$\therefore$ Kinetic energy at a distance $x = d$ will be $= \frac{1}{4\pi\varepsilon_o}\frac{qp}{d^2}$

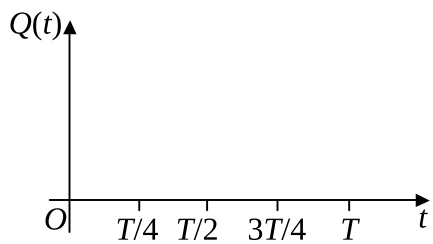
(b)

$$F = -\frac{dU}{dx}$$

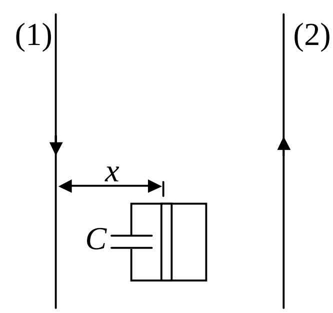
$$F = \frac{-2qp}{4\pi\varepsilon_o d^3}$$ (-ve sign implies an attractive force)

14. Two infinitely long parallel wires carrying currents $I = I_o \sin\omega t$ in opposite directions are placed a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown. Find the maximum current in the square loop. Also sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive. **[4]**





Sol. In the square loop magnetic field due to both the wires is out of paper.
For a elemental strip of thickness dx at a distance x from wire 1, magnetic field due to wire (1) and (2) will be



$$B = \frac{\mu_o i}{2\pi x} + \frac{\mu_o i}{2\pi(3a - x)}$$

Flux in the strip

$$d\Phi = B.dA = \frac{\mu_o i}{2\pi}\left[\frac{1}{x} + \frac{1}{3a - x}\right].adx$$

$$\therefore \quad \Phi = \frac{\mu_o ia}{2\pi}\left[\ln|x| - \ln(3a - x)\right]_a^{2a}$$

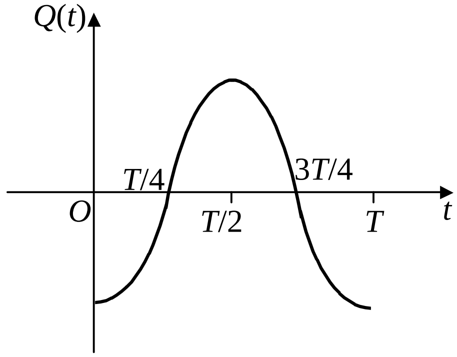
$$\Phi = \frac{\mu_o ai}{\pi}\sin\omega t.\ln(2)$$

$$E_{ind} = \left|\frac{d\Phi}{dt}\right| = \frac{\omega\mu_o ai\ln(2)\cos\omega t}{\pi}$$ Now $Q = CE_{ind}$ and $I = \frac{dQ}{dt}$

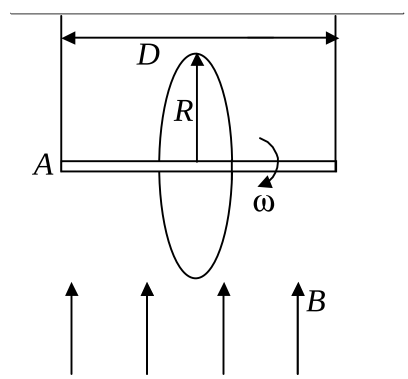
$$\therefore \quad i = \frac{C\omega^2\mu_o ai\ln(2)\sin\omega t}{\pi}$$

For $i_{\max}$, $\sin \omega t = 1$

$$i_{\max} = \frac{C\omega^2 \mu_o ai \ln(2)}{\pi}$$



15. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_o. Now a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_o/2$.



[4]

Sol. Let for ω string breaks; $2T_o = mg$, $T_o = mg/2$

current = $\frac{\omega}{2\pi} \times Q$

Magnetic moment = IA

$$\tau = M \times B = IAB$$

$$(T_1 - T_2)\frac{D}{2} = IAB$$

$$T_1 - T_2 = \frac{2IAB}{D}$$

$$T_1 + T_2 = mg$$

$$2T_1 = \frac{2IAB}{D} + mg$$

$$\frac{2 \times 3T_o}{2} = \frac{2\omega Q \times \pi R^2 \times B}{2\pi D} + mg$$

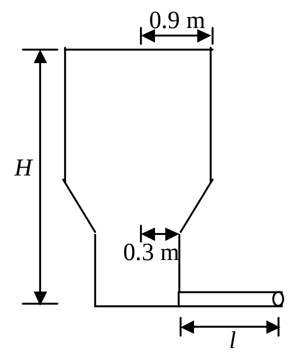
$$\frac{3mg}{2} = \frac{2\omega Q\pi R^2 \times B}{2\pi D} + mg$$

$$\frac{mg}{2} = \frac{\omega Q R^2 B}{D}$$

putting, $mg/2 = T_o$

$$\omega = \frac{D T_o}{Q R^2 B}$$

16. A liquid of density 900 kg/m^3 is filled in a cylindrical tank of upper radius 0.9 m and lower radius 0.3 m. A capillary tube of length l is attached at the bottom of the tank as shown in the figure. The capillary has outer radius 0.002 m and inner radius a. When pressure P is applied at the top of the tank volume flow rate of the liquid is 8×10^{-6} m^3/s and if capillary tube is detached, the liquid comes out from the tank with a velocity 10 m/s. Determine the coefficient of viscosity of the liquid.
[Given: $\pi a^2 = 10^{-6}$ m^2 and $a^2/l = 2 \times 10^{-6}$ m]



[4]

Sol. When capillary is not connected

$$P + P_o + \rho g H + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2 + P_o$$

$$p + \rho g H = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2 v_2}{A_1}$$

$$P + \rho g H = \frac{1}{2}\rho\left(v_2^2 - \left(\frac{A_2}{A_1}v_2^2\right)\right) = \frac{1}{2}\rho v_2^2\left(1 - \left(\frac{3}{a}\right)^2\right) = \frac{1}{2}\rho 100\left(1 - \frac{1}{a}\right) = \frac{800\rho}{18}$$

Excess pressure is $P + \rho_g W$

$$\Delta P = \frac{800\rho}{18}$$

$$\frac{dv}{dt} = \frac{\pi r^4 \Delta P}{8\eta l}$$

$$8 \times 10^{-6} = \frac{2 \times 10^{-6} \times 10^{-6} \times 800\rho}{8 \times 18 \times \eta}$$

$\eta = 1.25 \times 10^{-3}$ Ns/m^2

17. A string of mass per unit length μ is clamped at both ends such that one end of the string is at $x = 0$ and the other is at $x = l$. When string vibrates in fundamental mode, amplitude of the mid point of the string is a, and tension in the string is T. Find the total oscillation energy stored in the string.

[4]

Sol. The amplitude at a distance x from the origin is given by

$$A = a\sin kx$$

Considering an element of length dx of the string at a distance x from the origin.

The total energy of this element = its maximum kinetic energy

$$= \frac{1}{2}dm\omega^2A^2 = \frac{1}{2}\mu dx 4\pi^2 f^2 a^2 \sin^2 kx$$

$$= 2\pi^2\mu f^2 a^2 \sin^2 kxdx$$

Total energy of the string = $\int_0^L 2\pi^2\mu f^2 a^2 \sin^2 kxdx$

$$= \pi^2\mu f^2 a^2 \int_0^L (1-\cos 2kx)dx = \pi^2\mu f^2 a^2\left[x - \frac{\sin 2kx}{2}\right]_0^L$$

$$= \pi^2\mu f^2 a^2\left[L - \frac{\sin 2kL}{2}\right] = \pi^2\mu f^2 a^2 L$$

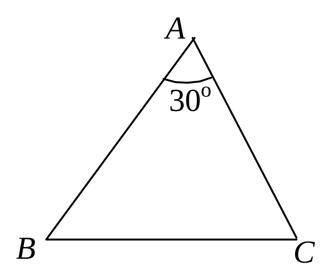
Since $\sin 2kL = \sin\frac{4\pi}{\lambda}\frac{\lambda}{2} = \sin 2\pi = 0$

$$= \pi^2\mu\frac{V^2}{\lambda^2}a^2L = \frac{\pi^2 L}{4L^2}a^2L = \frac{\pi^2a^2T}{4L}$$

18. A prism of refracting angle 30^o is coated with a thin film of transparent material of refractive index 2.2 on face AC of the prism. A light of wavelength 6600 Å is incident on face AB such that angle of incidence is 60^o, find

(a) the angle of emergence, and
[Given refractive index of the material of the prism is $\sqrt{3}$]

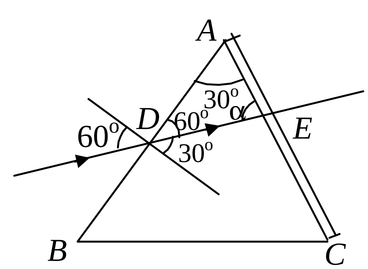
(b) the minimum value of thickness of the coated film on the face AC for which the light emerging from the face has maximum intensity.



[4]

Sol.(a) $1.\sin 60^o = \sqrt{3}\ .\ \sin r$

$\therefore \quad \sin r = \frac{1}{2} \quad \Rightarrow \quad r = 30^o$



In ΔADE

$$30 + (90 - 30) + \alpha = 180$$

$$\alpha = 90^o$$

$\therefore$ Ray refracted on the first surface of the prism is incident normally on the face AC.

Hence it will emerge undeviated.

$\therefore$ angle of emergence is zero.

(b) Intensity will be maximum if constructive interference takes place in the transmitted system.

$$2\mu t = n\lambda$$

$$t = \frac{\lambda}{2\mu} \qquad \text{(n = 1 for minimum thickness)}$$

$$t_{\min} = \frac{6000}{2\times 2.2} = 1500 \text{ Å}$$

19. Two point masses m_1 and m_2 are connected by a spring of natural length l_o. The spring is compressed such that the two point masses touch each other and then they are fastened by a string. Then the system is moved with a velocity v_o along positive x-axis. When the system reaches the origin the strings breaks (t = 0). The position of the point mass m_1 is given by

$x_1 = v_o t - A(1-\cos\omega t)$ where A and ω are constants.

Find the position of the second block as a function of time. Also find the relation between A and l_o.

[4]

Sol. Since there is no external force, momentum of the system is conserved.

$$\Rightarrow \quad m_1\frac{dx_1}{dt} + m_2\frac{dx_2}{dt} = (m_1+m_2)v_o$$

$$\Rightarrow \quad m_2\frac{dx_2}{dt} = (m_1+m_2)v_o - m_1\frac{dx_1}{dt}$$

$$= (m_1+m_2)v_o - m_1(v_o - A\omega\sin\omega t)$$

$$= m_2 v_o + m_1 a\omega\sin\omega t$$

$$\Rightarrow \quad \frac{dx_2}{dt} = v_o + \frac{m_1}{m_2}A\omega\sin\omega t$$

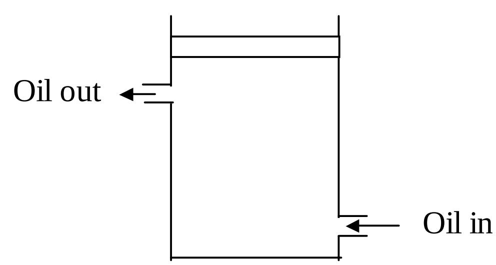
$$x_2 = \int_0^t v_o dt + \frac{m_1 A\omega}{m_2}\int_0^t \sin\omega t dt = v_o t + \frac{m_1}{m_2}(1-\cos\omega t)$$

$$x_2 - x_1 = \frac{m_1 A}{m_2}(1-\cos\omega t) + A(1-\cos\omega t)$$

Maximum value of $x_2 - x_1 = 2l_o$

$$\Rightarrow \quad 2l_o = \frac{2m_1 A}{m_2} + 2A = \frac{2(m_1+m_2)A}{m_2} \quad \Rightarrow \quad l_o = \frac{(m_1+m_2)A}{m_2}$$

20. The top of an insulated cylindrical container is covered by a disc having emissivity 0.6 and conductivity 0.167 W/K/m and thickness 1 cm. The temperature is maintained by circulating oil as shown.



[4]

(a) Find the radiation loss to the surroundings in $J/m^2/s$ if temperature of the upper surface of disc is 127 ^oC, and temperature of surroundings is 27^oC.

(b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection.

[Given $\sigma = \frac{17}{3} \times 10^{-8}$ $Wm^{-2}K^{-4}$].

20. (a) Rate of heat loss per unit area due to radiation

$$= e\sigma\left(T^4 - T_o^4\right) = 0.6 \times \frac{17}{3} \times 10^{-8}\left[(400)^4 - (300)^4\right]$$

$= 595\ J/m^2.\ s$

(b) Suppose temperature of oil is θ then rate of heat flow through conduction = rate of heat loss due to radiation.

$$\therefore \quad \frac{0.167 \times A(\theta - 127)}{1 \times 10^{-2}} = 595\ A \qquad [A = \text{area of the disc}]$$

After solving we get,

$\theta = 162.6\ ^oC$.

IIT-JEE 2003 Mains Questions & Solutions - Chemistry

(The questions are based on memory)

Break-up of questions:

Physical	Inorganic	Organic
8	**4**	**8**

1. Write the balanced chemical reactions involved in the extraction of lead from Galena. Mention oxidation state of lead in litharge.

[2]

Sol. The reactions involved in the extraction of lead from galena (PbS) are

$2PbS + 3O_2 \longrightarrow 2PbO + 2SO_2$

$PbS + 2O_2 \longrightarrow PbSO_4$

$2PbO + PbS \longrightarrow 3Pb + SO_2$

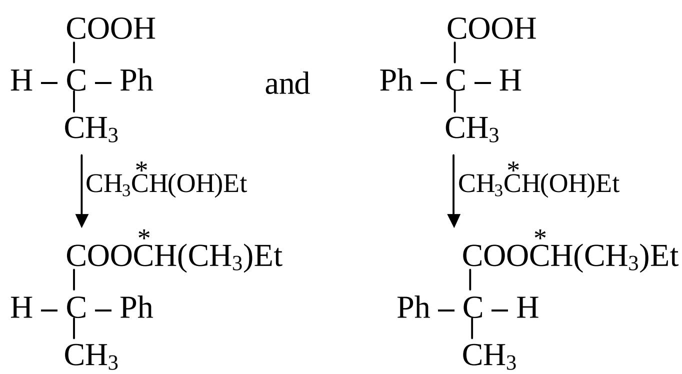
$PbSO_4 + PbS \longrightarrow 2Pb + 2SO_2$

Oxidation state of lead in litharge (PbO) is +2.

2. (±)2–phenylpropanoic acid on treatment with (+) 2–butanol gives (A) and (B). Deduce their structures and also establish stereochemical relation between them.

[2]

Sol. The two stereoisomers of 2–phenylpropanoic acid in the racemic mixture are



(A) (B)

(A) and (B) are diastereomers.

3. Find the molarity of water. Given: $\rho = 1000\ kg/m^3$

[2]

Sol. Let us consider 1 litre of water.

$\therefore$ Number of moles of solute present in 1 litre = $\frac{1000}{18}$ = 55.56 M

4. Name the Hetrogenous catalyst used in the polymerization of ethylene.

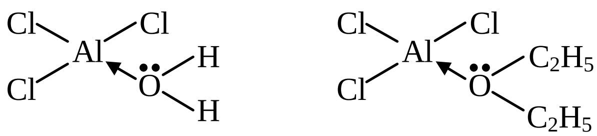
[2]

Sol. Zeigler Natta catalyst is a mixture of trialkyl aluminium and titanium tetrachloride which is used for the polymerization of ethylene.

5. Which of the two, anhydrous or hydrated $AlCl_3$ is more soluble in diethyl ether? Justify using the concepts of bonding in not more than 2 or 3 sentences.

[2]

Sol. Anhydrous $AlCl_3$ is more soluble in diethyl ether as the oxygen atom of the ether donates its pair of electrons to the vacant orbital of electron deficient aluminium of $AlCl_3$ through the formation of coordinate bond. But in case of hydrated $AlCl_3$ aluminium is not electron deficient as oxygen atom of water molecule has already donated its pair of electrons to meet the electron deficiency of aluminium.

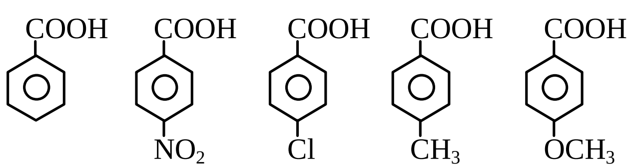


6. Match the following with their K_a values

Benzoic acid	4.2×10^{-5}
p–nitrobenzoic acid	3.3×10^{-5}
p–chlorobenzoic acid	6.4×10^{-5}
p–methylbenzoic acid	36.2×10^{-5}
p–methoxybenzoic acid	10.2×10^{-5}

[2]

Sol.



1 2 3 4 5

The correct order of acetic strength would be 2 > 3 > 1 > 4 > 5.

Therefore, the correct matching

Benzoic acid	6.4×10^{-5}
p–nitrobenzoic acid	36.2×10^{-5}
p–chlorobenzoic acid	10.2×10^{-5}
p–methylbenzoic acid	4.2×10^{-5}
p–methoxybenzoic acid	3.3×10^{-5}

7. The wavelength corresponding to maximum energy for hydrogen is 91.2 nm. Find the corresponding wavelength for He^+ ion.

[2]

Sol. For Maximum energy $n_1 = 1$ and $n_2 = \infty$

For H atom:

$$\frac{1}{\lambda_H} = R_H Z^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$\frac{1}{\lambda_H} = R_H Z_H^2 \quad \ldots(i)$$

For He^+ ion:

$$\frac{1}{\lambda_{He^+}} = R_{He} \times Z_{He^+}^2 \quad \ldots(ii)$$

Dividing equation (i) by (ii), we get

$$\frac{\lambda_{He^+}}{\lambda_H} = \frac{R_H Z_H^2}{R_{He^+} Z_{He^+}^2} \quad (R_H = R_{He^+})$$

$$\lambda_{He^+} = \frac{\lambda_H}{4} = \textbf{22.8 nm}.$$

8.

$$HOOC-(CH_2)_2-\underset{NH_2}{\underset{|}{CH}}-COOH \quad ; \quad NH_2-(CH_2)_4-\underset{NH_2}{\underset{|}{CH}}-COOH$$

Find the structure of possible two dipeptides.

[2]

Sol. The structures of two possible dipeptides are

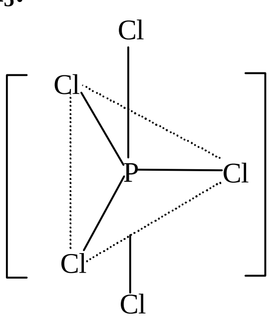
$$HOOC-(CH_2)_2-\underset{\underset{\oplus}{NH_3}}{\underset{|}{CH}}-CO-NH-\underset{COO^-}{\underset{|}{CH}}-(CH_2)_4-NH_2$$

$$HOOC-(CH_2)_2-\underset{COO^-}{\underset{|}{CH}}-NH-CO-\underset{\underset{\oplus}{NH_3}}{\underset{|}{CH}}-(CH_2)_4-NH_2$$

9. Using VSEPR theory deduce the structures of PCl_5 and BrF_5.

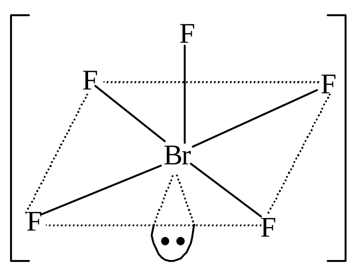
[2]

Sol. **PCl_5:**



Triangular bipyramidal

BrF_5:



Square pyramidal

10. The average velocity of a gas is 400 m s^{-1}, find the rms velocity of the gas.

[2]

Sol. $C_{av} = \sqrt{\frac{8RT}{\pi M}}$ ...(i)

$C_{rms} = \sqrt{\frac{3RT}{M}}$ ...(ii)

$\therefore \frac{C_{rms}}{C_{av}} = \sqrt{\frac{3RT}{M} \times \frac{\pi M}{8RT}} = \sqrt{\frac{3\pi}{8}}$

$\therefore C_{rms} = \sqrt{\frac{3\pi}{8}} \times C_{av}$

$= \sqrt{\frac{3 \times 3.14}{8}} \times 400$ = **434.05 ms^{-1}**

11. (a) Will pH value of water be same at temperature 25°C and 4°C. Justify in not more than 2 or 3 sentences.

[2]

(b) Two students make Daniel cells in laboratory. They take $ZnSO_4$ from common stock with Cu as positive electrode. The emf of one cell is 0.03 V more than the other. The concentration of $CuSO_4$ in cell of higher emf is 0.5 M. Find the concentration of $CuSO_4$ in second cell.

Given: $\frac{2.3\ RT}{F} = 0.06\,V$

[2]

Sol.(a) pH of solution depends upon H^+ ion concentration, which depends on K_w which is a function of temperature. Therefore, change in temperature brings a change in pH value for given sample of water.

(b) Let the emf of 1^{st} cell be x volt.
Emf of 2^{nd} cell = (x + 0.03) V
$[Cu^{2+}]$ in 2^{nd} cell = 0.5 M
$[Cu^{2+}]$ in 1^{st} cell = ?

$E_1 = E_1^o - \frac{2.303RT}{2F} \log \frac{[Zn^{2+}]}{[Cu^{2+}]_1}$

$E_2 = E_1^o - \frac{2.303RT}{2F} \log \frac{[Zn^{2+}]}{[Cu^{2+}]_2}$

$E_2 - E_1 = E_1^o - \frac{2.303RT}{2F} \log \frac{[Zn^{2+}]}{[Cu^{2+}]_2} - E_1^o + \frac{2.303RT}{2F} \log \frac{[Zn^{2+}]}{[Cu^{2+}]_1}$

$$x + 0.03 - x = \frac{2.303RT}{2F}\left[\log\frac{[Zn^{2+}]}{[Cu^{2+}]_1} - \log\frac{[Zn^{2+}]}{[Cu^{2+}]_2}\right]$$

$$0.03 = 0.03\left[\log\frac{[Zn^{2+}]}{[Cu^{2+}]_1}\times\frac{[Cu^{2+}]_2}{[Zn^{2+}]}\right]$$

$$1 = \log\frac{[Cu^{2+}]_2}{[Cu^{2+}]_1}$$

$$1 = \log\frac{0.5}{[Cu^{2+}]_1}$$

$[Cu^{2+}]_1$ = **0.05 M**

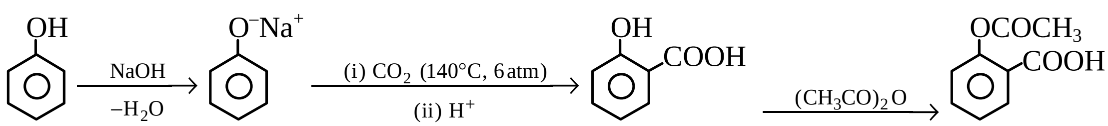
12. Carry out the following conversions.

(i) Phenol to Aspirin

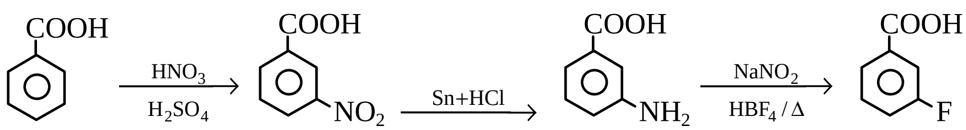
(ii) Benzoic acid to meta–fluorobenzoic acid in not more than three steps.

[4]

Sol.(i)



(ii)



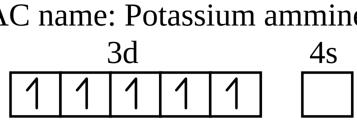
13. Write the IUPAC name of the compound $K_2[Cr(NO)(CN)_4(NH_3)]$. Spin magnetic moment of the complex, μ = 1.73 BM. Give the structure of anion.

[4]

Sol. The spin magnetic moment, μ of the complex is 1.73 BM. It means that nucleus of the complex, chromium ion has one unpaired electron. So the ligand NO is unit positively charged.

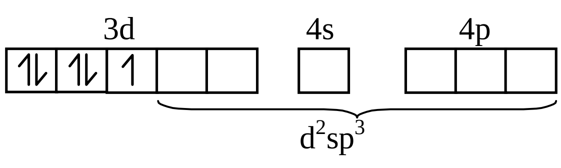
IUPAC name: Potassium amminetetracyanonitrosoniumchromate (I).

Cr^+:

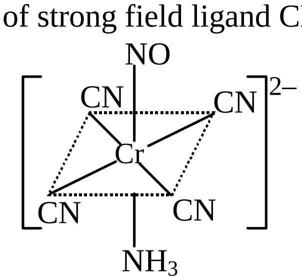


Electronic configuration of Cr^+ under the influence of strong field ligand CN^- is

Cr^+:



;



So,

Hybridization: d^2sp^3
Shape: Octahedral

14. A salt mixture consists of a yellow solid (A) and a colourless solid (B). The aqueous solution of the mixture

(i) On passing H_2S, we get a black precipitate of (C), which dissolves only in aqua regia. On extraction and reaction with $SnCl_2$ a grayish white precipitate is obtained.

(ii) On treatment with ammonium hydroxide a reddish brown precipitate (D) is obtained.

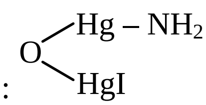
The sodium extract of the solution gives the following tests

(i) On reaction with $AgNO_3$ it gives a yellow precipitate which is insoluble in NH_3.

(ii) On shaking with $FeCl_3$ and CCl_4 a violet colouration in CCl_4 layer is obtained.

Mixture on performing flame test gives lilac colour. Identify the compounds (A), (B), (C) and (D).

[4]

Sol. (A): HgI_2 (B): KI (C): HgS (D): 

15. (a) For He molecule C_v value is 3/2 R, independent of temperature. But for H_2 at very low temperature 3/2 R, at moderate temperature 5/2 R and at higher temperature > 5/2 R. Explain the temperature dependence and justify.

[2]

(b) Consider the three solvents of identical molar masses. Match their boiling point with their K_b values:

Solvents	Boiling point	K_b values
X	100°C	0.92
Y	27°C	0.63
Z	283°C	0.53

[2]

Sol.(a) Helium molecule is monoatomic so it has just three degrees of freedom corresponding to the three translational motion at all temperature and hence C_v value is always 3/2 R.

Hydrogen molecule is diatomic whose atoms are not rigidly held so they vibrate about a well defined average separation. For hydrogen molecule we have rotational and vibrational motion both besides translational motion. These two additional contributions increase its total heat capacity. Contribution from vibrational motion is not appreciable at low temperature but increases from 0 to R on raising temperature.

(b) Higher value of K_b of a solvent suggests larger polarity of solvent molecules which in turn implies higher boiling point due to dipole–dipole interaction. Therefore, the correct order of K_b values of the three given solvents is

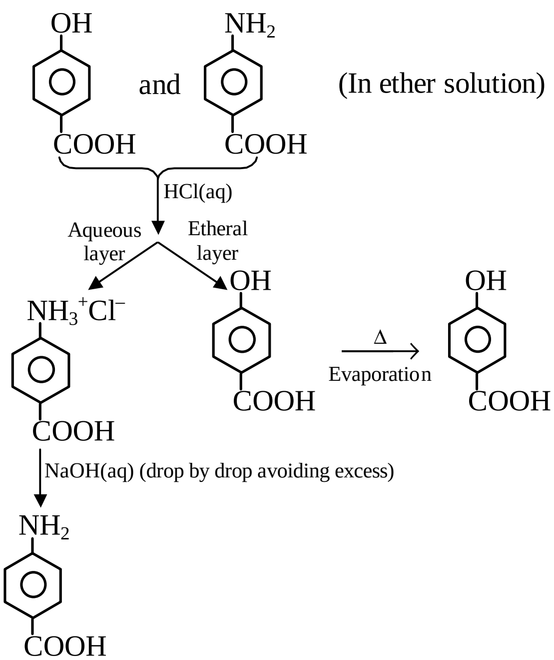
Solvents	Boiling point	K_b values

X	100°C	0.63
Y	27°C	0.53
Z	283°C	0.92

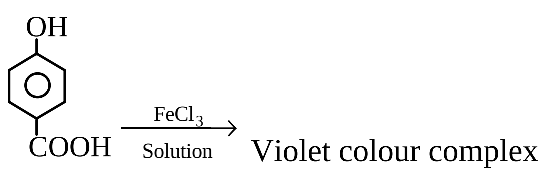
16. You have an ether solution containing 4–hydroxybenzoic acid and 4–aminobenzoic acid. Explain how will you separate the two in not more than 3 steps. Give confirmatory tests with reagents and conditions for functional groups of each.

[4]

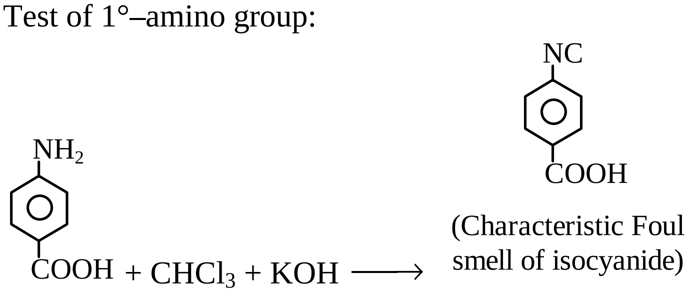
Sol.



Test of Phenolic group:



Test of 1°–amino group:



Test of Carboxylic group:

$$-COOH \xrightarrow{NaHCO_3} CO_2\uparrow \xrightarrow[\text{lime water}]{\text{With effervesence}} \text{Milky solution}$$

17. (A), $C_6H_{12} \xrightarrow{HCl}$ (B), $C_6H_{13}Cl$ + (C), $C_6H_{13}Cl$

(B) $\xrightarrow{\text{Alcoholic KOH}}$ (D) (an isomer of (A))

(D) $\xrightarrow{\text{Ozonolysis}}$ (E) (Positive iodoform and negative Fehling's solution test)

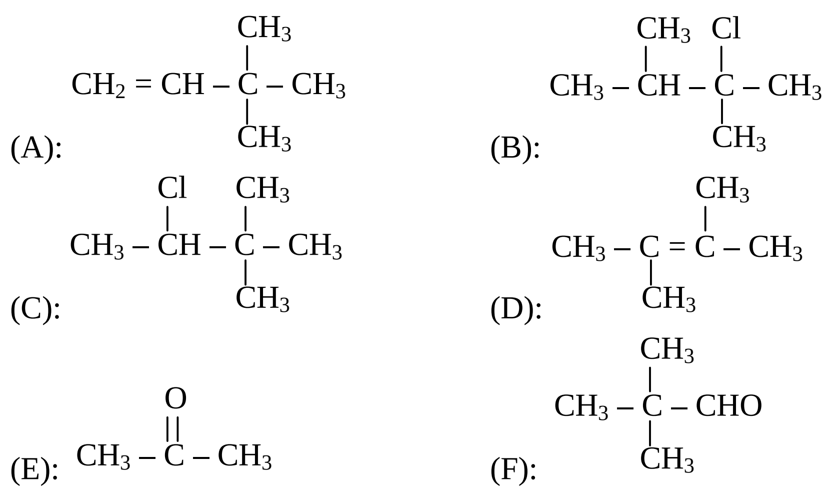
(A) $\xrightarrow{\text{Ozonolysis}}$ (F) + (G) (Positive Tollen's test for both)

(F) + (G) $\xrightarrow{\text{conc. NaOH}}$ HCOONa + A primary alcohol

Identify the compounds (A) to (D).

[4]

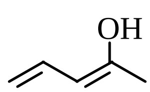
Sol.



(G): HCHO

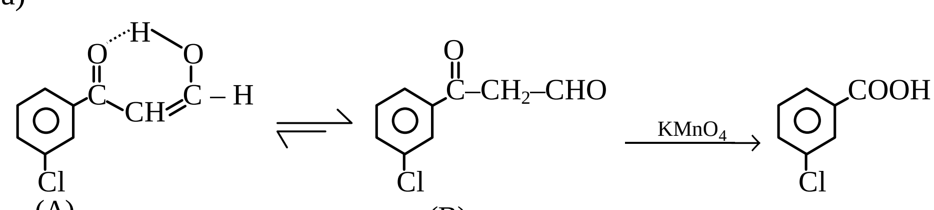
18. (a) A compound $C_9H_7O_2Cl$ exists predominantly in enol form (A) and also in keto form (B). On oxidation with $KMnO_4$ it gives m–chlorobenzoic acid as one of the products. Identify the compounds (A) and (B).

[2]

(b) Draw the resonating structures of .

[2]

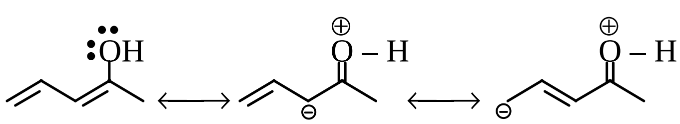
Sol.(a)



(A) (B)

$C_9H_7O_2Cl$

(b)



19. (a) Marbles of diameter 10 mm are to be put in a square area of side 40 mm so that their centers are within this area. Find the maximum number of marbles per unit area and deduce an expression for calculating it.

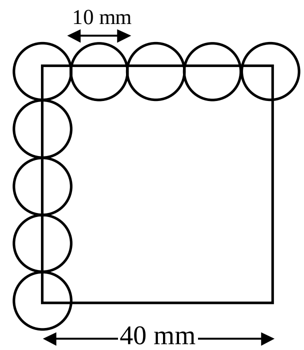
[2]

(b) In a solution of 100 ml 0.5 M acetic acid, one g of active charcoal is added, which adsorbs acetic acid. It is found that the concentration of acetic acid becomes 0.49 M. If surface area of charcoal is 3.01×10^2 m^2, calculate the area occupied by single acetic acid molecule on surface of charcoal.

[2]

Sol.(a)

Side of a square = 40 mm
Diameter of a marbles = 10 mm
Number of marbles along an edge of the square with their centers within the square is equal to 5.
Maximum number of marbles per unit area $= 5 \times 5 = 25$



(b) Number of moles of acetic acid in 100 ml before adding charcoal = 0.05
Number of moles of acetic acid in 100 ml after adding charcoal = 0.049
Number of moles of acetic acid adsorbed on the surface of charcoal = 0.001
Number of molecules of acetic acid adsorbed on the surface of charcoal

$= 0.001 \times 6.02 \times 10^{23} = 6.02 \times 10^{20}$

Surface area of charcoal = 3.01×10^2 m^2
Area occupied by single acetic acid molecule on the surface of charcoal

$$= \frac{3.01\times10^2}{6.02\times10^{20}} = 5 \times 10^{-19}\ m^2$$

20. $Na_2CO_3 \xrightarrow{SO_2} (A) \xrightarrow{Na_2CO_3} (B) \xrightarrow[\text{Sulphur}/\Delta]{\text{Elemental}} (C) \xrightarrow{I_2} (D)$

Identify the compounds (A), (B), (C), (D) and give oxidation state of sulphur in each compound.

[4]

Sol. (A): $NaH\overset{+4}{S}O_3$ (B): $Na_2\overset{+4}{S}O_3$ (C): $Na_2\overset{+2}{S_2}O_3$ (D): $Na_2\overset{+2.5}{S_4}O_6$